MATHS SAMPLE PAPER

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:

1. It consists of two sections- I and II
2. Section I has 16 questions. Internal choice is provided in questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:

1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided.

PART-A

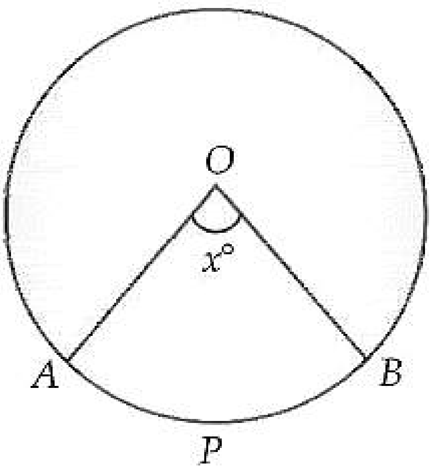
Section-I

1. Write the value of $\cot^2\theta - \frac{1}{\sin^2\theta}$

OR

If $k + 1 = \sec^2\theta (1 + \sin \theta)(1 - \sin \theta)$, then find the value of k.

2. In given fig., O is the center of a circle. If the area of the sector OAPB is 5/36 times the area of the circle, then find the value of x



3. If the areas of three adjacent faces of a cuboid are X, Y, and Z respectively, then find the volume of cuboid in terms of X, Y and Z.

4. Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective?

5. If $\cos \theta = 0.6$, then $5\sin\theta - 3\tan\theta$ = ______.

6. Write one rational and one irrational number lying between 0.25 and 0.32.

OR

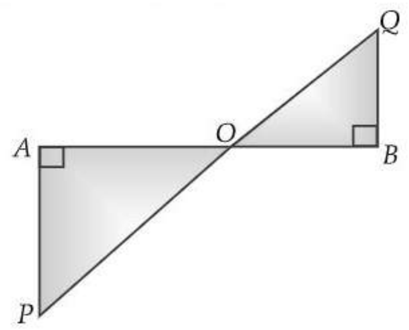
Explain why 13233343563715 is a composite number?

7. If R (5, 6) is the midpoint of the line segment joining the points A (6, 5) and B (4, y) then find the value of 'y'.

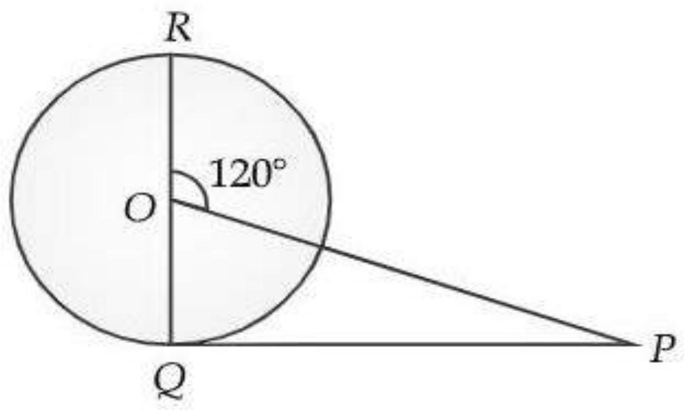
8. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3DE. Then, what can you say about the similarity and congruency of these triangles.

OR

In the given figure, if $\angle A = 90°$, $\angle B = 90°$, OB = 4.5cm, OA = 6 cm and AP = 4 cm, then QB ....



9. PQ is a tangent drawn from an external point P to a circle with centre O and QOR is the diameter of the circle. If ∠POR = 120°, the measure of ∠OPQ.



10. Calculate the largest number which divides 70 and 125, leaves remainders 5 and 8, respectively.

11. Find the zeroes of the quadratic polynomial $x^2 + 99x + 127$.

OR

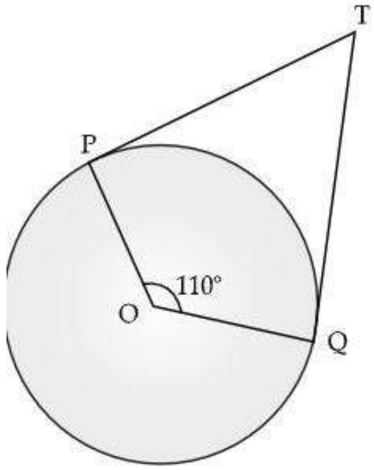
For what value of k, the roots of the equation $x^2 + 4.x + k = 0$ are real?

12. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1= 0$ have no solution. Justify?

13. The n^{th} term of an A.P. –1, 4, 9, 14.... is 129. Find the value of n?

14. If the distance between the points (4, p) and (1, 0) is 5, then find the value of p.

15. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then find ∠PTQ.



16. Find the volume of a right circular cylinder with base radius 7 cm and height 10 cm. (Use $\pi = {}^{22}/_{7}$)

Section II

17. Case Study – I: Jaspal takes a loan from a bank for his car. Jaspal Singh repays his total loan of Rs.118000 by paying every month starting with the first instalment of Rs 1000. He increases the installment by Rs.100 every month.

(i) If the given problem is based on AP, then what is the first term and common difference?
A. 100, 100
B. 1000, 100
C. 1000, 1000
D. None of these

(ii) In how many months the loan will be cleared?
A. 20
B. 30

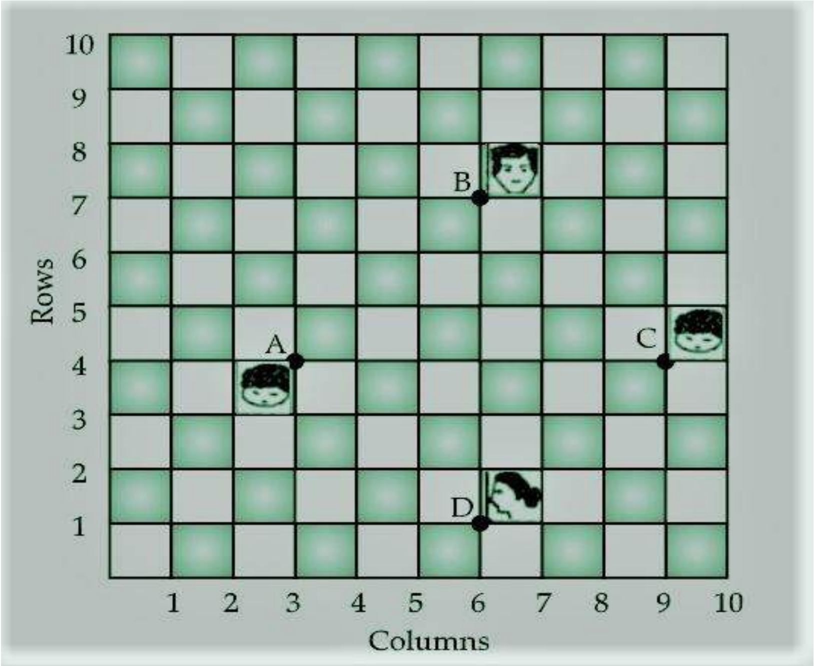
C. 40
D. 50

(iii) The amount paid by him in 30th installment is
A. 3900
B. 3500
C. 3000
D. 3600

(iv) The amount paid by him in 30 installments is
A. 37000
B. 73500
C. 75300
D. 53700

(v) What amount does he still have to pay after 30th installment?
A. 45500
B. 44000
C. 54500
D. 44500

18. Case Study - II: In a room, 4 friends are seated at the points A, B, C and D as shown in figure. Reeta and Meeta walk into the room and after observing for a few minutes Reeta asks Meeta.



(i) What is the position of A ?

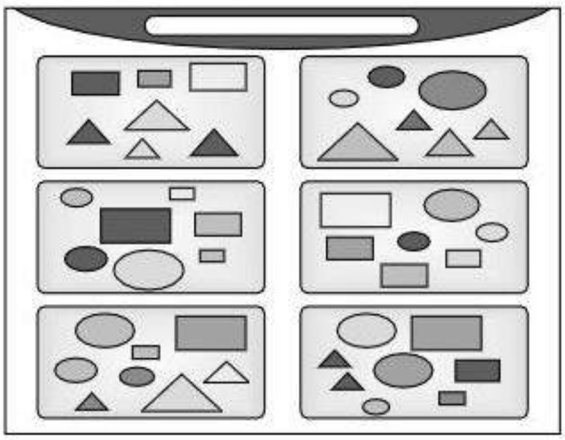
A. (4, 3)
B. (3, 3)
C. (3, 4)
D. None of these

(ii) What is the middle position of B and C?
A. $\left(\frac{15}{2}, \frac{11}{2}\right)$
B. $\left(\frac{2}{15}, \frac{11}{2}\right)$
C. $\left(\frac{1}{2}, \frac{1}{2}\right)$
D. None of these

(iii) What is the position of D ?
A. (6, 0)
B. (0, 6)
C. (6, 1)
D. (1, 6)
(iv) What is the distance between A and B?
A. $3\sqrt{2}$
B. $2\sqrt{3}$
C. $2\sqrt{2}$
D. $3\sqrt{3}$

(v) What is the distance between C and D?
A. $\sqrt{18}$
B. $\sqrt{14}$
C. $\sqrt{15}$
D. $\sqrt{17}$

19. Case Study – III: A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random.



(i) How many triangles are of red colour and how many squares are of blue colour?
A. 5, 4
B. 4, 5
C. 5, 5
D. 5, 6

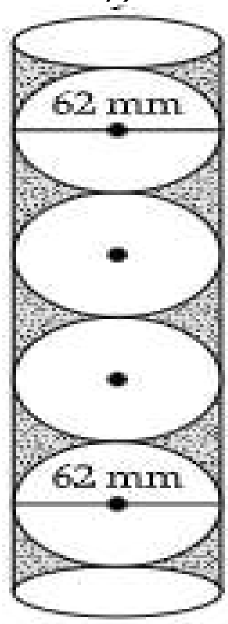
(ii) Find the probability that lost piece is triangle.
A. 4/9
B. 5/9
C. 1/3
D. 5/18

(iii) Find the probability that lost piece is square.
A. 4/9
B. 5/9
C. 1/3
D. 5/18

(iv) Find the probability that lost piece is square of blue color
A. 4/9
B. 5/9
C. 1/3
D. 5/18

(v) Find the probability that lost piece is triangle of red color
A. 4/9
B. 5/9
C. 1/3
D. 5/18

20. **Five** tennis balls, diameter 62 mm are placed in cylindrical card tubes (figure for visualization only)



(i) Find the radius of the tennis balls
A. 30 mm
B. 29 mm
C. 31 mm
D. 32 mm

(ii) Volume of 1 ball is equal to
A. 125 cm^3
B. 123.5 cm^3
C. 120.30 cm^3
D. 124.84 cm^3

(iii) Find the height of the tube
A. 300 mm
B. 320 mm
C. 310 mm
D. 301 mm

(iv) Find the volume of the tube
A. 963 cm^3
B. 966.3 cm^3
C. 939.23 cm^3
D. 936.29 cm^3

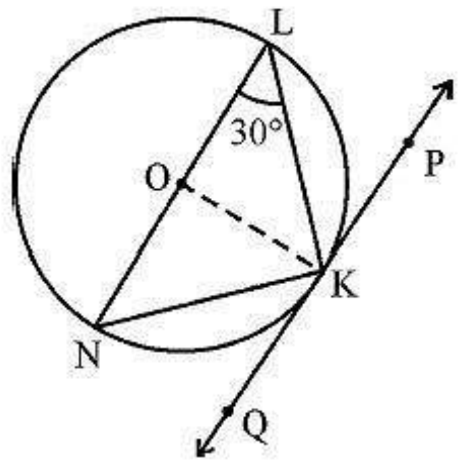
(v) Find the volume of unfilled space (shaded area) in the tube.
A. 310.9 cm^3

B. 312.09 cm^3
C. 301.90 cm^3
D. 321.09 cm^3

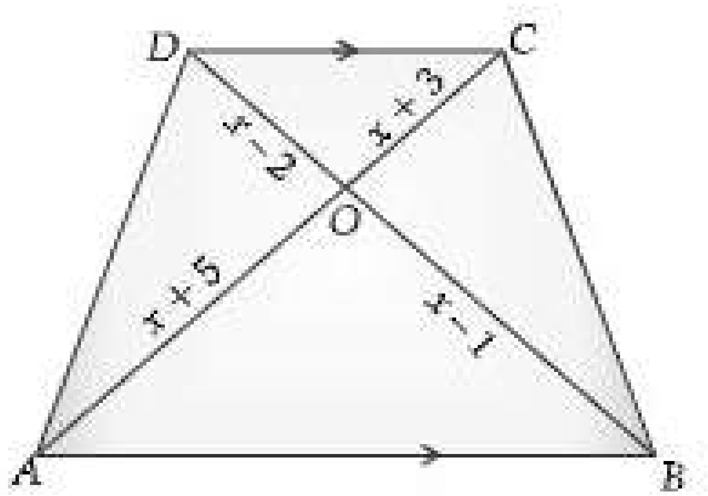
PART–B

Section III

21. In Figure, O is the center of the circle and LN is a diameter. If PQ is a tangent to the circle at K and ∠KLN = 30°, find ∠PKL.



22. In the given figure, if AB || DC, find the value of x.



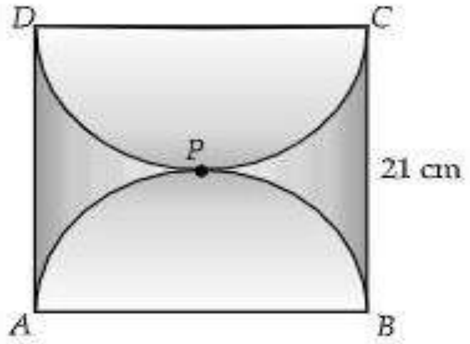
23. Prove that: $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

24. Manu is 1.7 m tall, he wants to see the light tower near his house and measure its height. The distance of tower from his house is $20\sqrt{3}$ m . So he used the concept of trigonometry. The angle of elevation from the eye of Manu to the top of tower is 30°. Find the height of the tower.

OR

The tops of two towers of height x and y, standing on the ground, subtend the angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y.

25. Find the perimeter of the shaded region if ABCD is a square of side 21 cm and APB and CPD are 2 semicircles. (use π = 22/7)



26. Find the mean of children per family from the data given below

Number of children	0	1	2	3	4	5
Number of families	5	11	25	12	5	2

Section IV

27. If α, β and γ are zeros of the polynomial $6x^3 + 3x^2 - 3x + 1$, then find the. value of $\alpha^{-1} + \beta^{-1} + y^{-1}$.

OR

Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

28. Find the value(s) of k so that the pair of equations x + 2y = 5and 3x + ky + 15 = 0 has a unique solution.

29. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280.Find the sum of its first n terms.

30. If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C. Prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

31. Prove that $\sin\theta\,(1+\tan\theta) + \cos\theta\,(1 + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$

32. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

33. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh wins the game if all the tosses give the same result (i.e. three heads or three tails) and loses otherwise. Find the probability of Ramesh losing the game.

Section V

34. Solve the following system of equations

$21/x + 47/y = 110$

$47/x + 21/y = 162,\ x, y \neq 0$

OR

If the roots of the quadratic equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

35. From a rectangular block of wood, having dimensions 15 cm × 10 cm × 3.5 cm a pen stand is made by making four conical depressions. The radius of each one of the depression is 0.5 cm and the depth is 2.1 cm. Find the volume of wood left in the penstand.

36. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB.

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Hints and Solutions

1. -1 OR k = 0

2. 50°

3. $\sqrt{xyz}$

4. 47/50

5. 0

6. 0.3 and 0.29873142…… (Answers may vary) OR The given number is divisible by 3, thus it has more than 2 factors, hence it is composite.

7. y = 7

8. The triangles are similar by AA similarity but not congruent. OR QB = 3 cm.

9. ∠OPQ = 30°

10. 13

11. $\frac{-99\pm\sqrt{9293}}{2}$

12. $\frac{-1}{3} = \frac{-2}{6} \neq \frac{5}{1}$ The condition for parallel lines.

13. n = 27

14. p = 4, -4

15. ∠PTQ = 70°

16. 1540 cm^3

17. (i) B) 1000, 100

(ii) C) 40

(iii) A) 3900

(iv) B) 73500

(v) D) 44500

18. (i) C) (3, 4)

(ii) A) (15/2, 11,2)

(iii) C) (6, 1)

(iv) A) $3\sqrt{2}$

(v) A) $\sqrt{18}$

19. (i) D) 5, 6

(ii) A) 4/9

(iii) B) 5/9

(iv) C) 1/3

(v) D) 5/18

20. (i) C) 31 mm

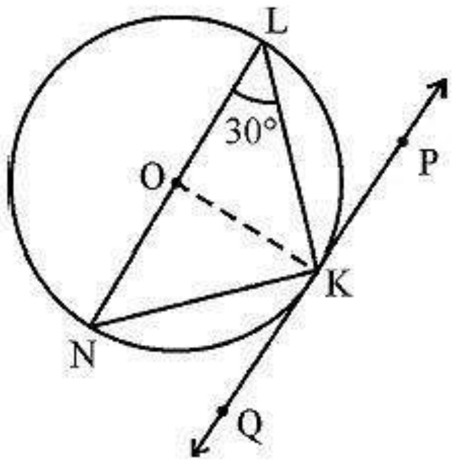
(ii) D) 124.84 cm^3

(iii) C) 310 mm

(iv) D) 936.29 cm^3

(v) B) 312.09 cm^3

21.

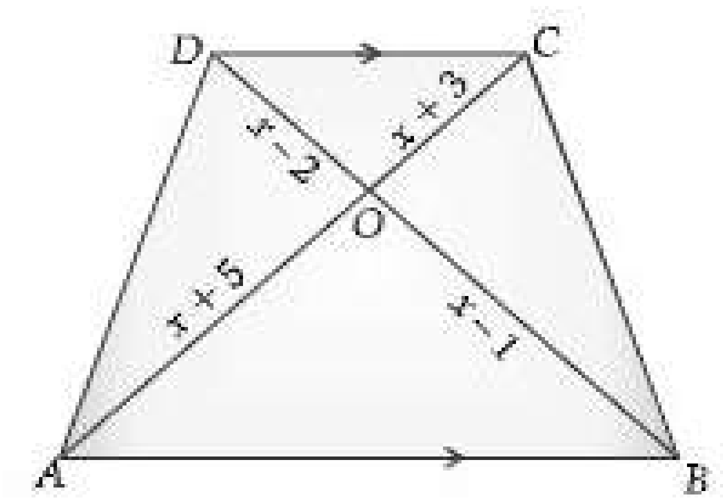


$\angle$LKN = 90° (Angles in a semicircle)

$\angle$LNK = 60° (Angle sum property)

$\angle$LKP = $\angle$LNK = 60° (Alternate segment theorem)

22.



$$\frac{OA}{OC} = \frac{OB}{OD}$$

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

x = 7

23. To Prove: $(\sin A + \operatorname{cosec} A)^2 (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Proof:
LHS = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$
Use the formula $(a+b)^2 = a^2 + b^2 + 2ab$ to get,

= $(\sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A) + (\cos^2 A + \sec^2 A + 2\cos A \sec A)$
Since sinθ=1/cosecθ and cosθ = 1/secθ

$$= \left(sin^2 A + cosec^2 A + 2\, sinA \frac{1}{sinA} \right) + \left(cos^2 A + sec^2 A + 2\, cosA \frac{1}{cosA} \right)$$

= $\sin^2 A + \operatorname{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$
=$(\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2$
Use the identities $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$ and $\operatorname{cosec}^2 A = 1 + \cot^2 A$ to get

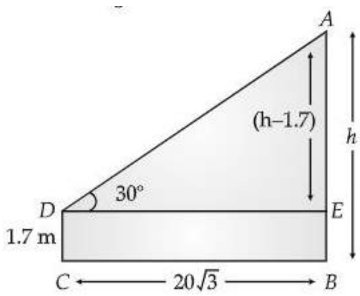
= $1 + 1 + \tan^2 A + 1 + \cot^2 A + 2 + 2$

= $1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A$

= $7 + \tan^2 A + \cot^2 A$

= RHS

24.



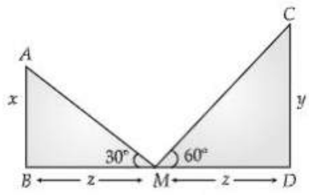
In △AED,

$$\frac{AE}{ED} = \tan 30^o$$

$$\frac{AE}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Height of tower = AB = AE + BE = 20 + 1.7 = 21.7 m

OR



$$\frac{x}{z} = \tan 30^o$$

$$\frac{y}{z} = \tan 60^o$$

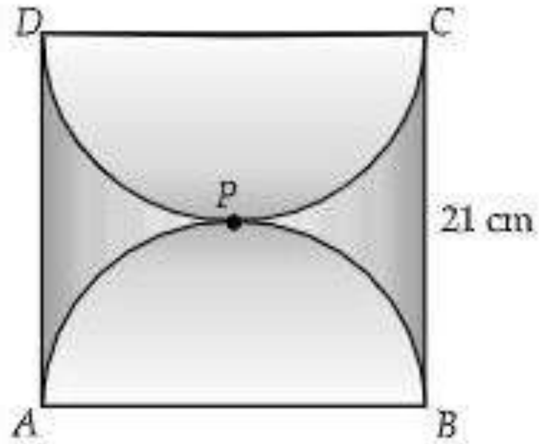
$$\frac{x}{z} = \frac{1}{\sqrt{3}}$$

$$\frac{y}{z} = \sqrt{3}$$

$$\frac{\frac{x}{z}}{\frac{y}{z}} = \frac{x}{y} = \frac{1}{3}$$

x : y = 1 : 3

25.



Perimeter of the shaded region = AD + DPC (Arc) + CB + BPA (Arc)

AD = CB = 21 cm

Radius of the semicircle = 21/2 cm

Length of arc DPC = BPA = $\pi r = \frac{22}{7} \times \frac{21}{2}$ = 33 cm

Perimeter of the shaded region = 21 + 33 + 21 + 33 = 108 cm

26.

Number of Children(x_i)	Number of Families(f_i)	f_ix_i
0	5	0
1	11	11
2	25	50
3	12	36
4	5	20
5	2	10
Total	$\sum f_i = 60$	$\sum f_ix_i = 127$

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{127}{60} = 2.11$

NOTE - The data in decimals is for calculation.

For a real survey (Not in examination) the answer could have been rounded off to 2 (Approx) for realistic purposes.

27. Sum of zeroes : $\alpha + \beta + \gamma = -b/a = -3/6 = -1/2$

Sum of zeroes taken two at a time : $\alpha\beta + \beta y + \gamma\alpha = c/a = -3/6 = -1/2$

And Product of zeroes $\alpha\beta\gamma = -d/a = -1/6$

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}=\frac{\frac{-1}{2}}{\frac{-1}{6}}=3$$

OR

Sum of zeros:

$\alpha + \beta = 3/2$

Product of zeros

$\alpha\beta = ½$

$3\alpha + 3\beta = 3(\alpha + \beta) = 3 \times 3/2 = 9/2$

$3\alpha \times 3\beta = 9\alpha\beta = 9/2$

New quadratic polynomial whose zeros are 3α and 3β is

x^2 – (Sum of the roots) x + Product of the roots

$x^2 - (9/2)x + 9/2$

$2x^2 - 9x + 9$

28. For unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{1}{3} \neq \frac{2}{k}$$

$k \neq 6$

For all values of k except k = 6, the given pair of linear equations in 2 variables will have a unique solution.

29. $S_4 = 40$ and $S_{14} = 280$

$\frac{4}{2}[2a+3d]=40$ and $\frac{14}{2}[2a+13d]=280$

2a + 3d = 20 ..[1]

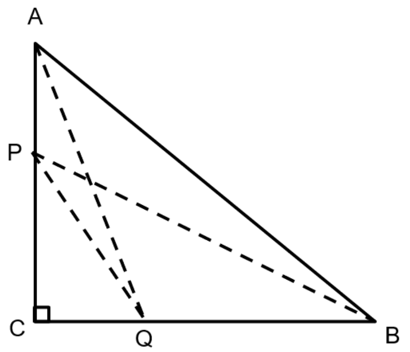
2a + 13d = 40 ..[2]

Solving equations [1] and [2],

a = 7, d = 2

$$S_n = \frac{n}{2}[2(7) + (n-1)(2)] = n^2 + 6n$$

30.



$AQ^2 = AC^2 + CQ^2$ (By Pythagoras theorem)

$BP^2 = PC^2 + BC^2$

Add the above statements,

$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2 = AC^2 + BC^2 + CQ^2 + PC^2$

$= AB^2 + PQ^2$

= RHS

Hence Proved

31. Consider LHS

$\sin\theta\,(1+\tan\theta) + \cos\theta\,(1 + \cot\theta)$

$\sin\theta\,(1+\frac{\sin\theta}{\cos\theta}) + \cos\theta\,(1 + \frac{\cos\theta}{\sin\theta})$

$\sin\theta\,(\frac{\cos\theta+\sin\theta}{\cos\theta}) + \cos\theta\,(\frac{\sin\theta+\cos\theta}{\sin\theta})$

$(\sin\theta + \cos\theta)(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta})$

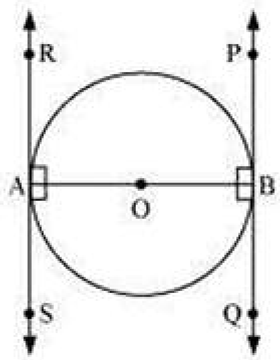
$(\sin\theta + \cos\theta)(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta})$ ..$[\sin^2\theta + \cos^2\theta = 1]$

$$\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$$

$\sec\theta + \operatorname{cosec}\theta$ = RHS

Hence Proved

32.



Let AB is diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively. Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ RS and OB ⊥ PQ

∠OAR = 90°

∠OAS = 90°

∠OBP = 90°

∠OBQ = 90°

It can be observed that

∠OAR = ∠ OBQ (Alternate interior angles)

∠OAS = ∠ OBP (Alternate interior angles)

Since alternate interior angles are equal, lines PQ and RS will be parallel.

33. Since the one-rupee coin is tossed thrice,

Total number of outcomes = 2^3 = 8

The favourable outcomes of Ramesh winning = {HHH, TTT} = 2

P(Win) = $\frac{2}{8} = \frac{1}{4}$

Thus, P(Lose) = $1 - \frac{1}{4} = \frac{3}{4}$

34. 21/x + 47/y = 110

47/x + 21/y = 162, x, y ≠ 0

Let 1/x = A and 1/y = B

21A + 47B = 110

47A + 21B = 162

Add and subtract the above equations,

68A + 68B = 272

A + B = 4 ..[1]

-26A + 26B = -52

-A + B = -2

A – B = 2 ..[2]

Add Equations [1] and [2]

2A = 6

A = 3 and B = 1

Thus 1/x = 3 and 1/y = 1

x = 1/3 and y = 1

OR

We know that if discriminant = 0, the roots of the equation are equal.

$$\left(-2\left(a^2-bc\right)\right)^2=4\left(c^2-ab\right)\left(b^2-ac\right)$$

$$4\left(a^4+b^2c^2-2a^2bc\right)=4\left(b^2c^2+a^2bc-ab^3-ac^3\right)$$

$$a^4+b^2c^2-2a^2bc=b^2c^2+a^2bc-ab^3-ac^3$$

$$a^4+b^2c^2-2a^2bc-b^2c^2-a^2bc+ab^3+ac^3=0$$

$$a\left(a^3+b^3+c^3\right)=3a^2bc$$

$$a\left(a^3+b^3+c^3-3abc\right)=0$$

$$a=0 \text{ or } a^3+b^3+c^3-3abc=0$$

$$a=0 \text{ or } a^3+b^3+c^3=3abc$$

35.

Volume of cuboidal block = l x b x h

$= 15 \times 10 \times 3.5 = 525\ cm^3$

Volume of one cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1\ cm^3$

$= 0.55\ cm^3$

Volume of 4 cones

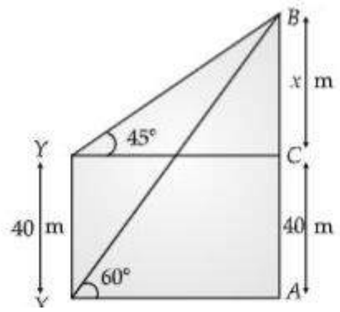
$= 0.55 \times 4 = 2.2\ cm^3$

Volume of wood remaining in pen stand

$= 525 - 2.2$

$= 522.80\ cm^3$.

36.



In ΔBCY,

$\frac{BC}{CY} = \tan 45^o$

BC = CY = x

Also, By symmetry, CY = AX

In Δ BAX,

$\frac{BA}{AX} = \tan 60^o$

$\frac{x + 40}{x} = \sqrt{3}$

x = $20\sqrt{3}$ + 20 m

Thus, height of the tower AB = 60 + $20\sqrt{3}$ = 20(3 + $\sqrt{3}$) m

Also,

$$\frac{AX}{BX} = \cos 60^o$$

$$\frac{20\sqrt{3} + 20}{BX} = \frac{1}{2}$$

BX = $40\sqrt{3}$ + 40 = $40(\sqrt{3} + 1)$ m

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